**P2.** Assume a particle moves along the path given by r(t) = (cos^2(t), cos(t)), t in R+, where t corresponds to time. Sketch the path. What is the shape of the path? Find the points where the velocity is zero and find the acceleration at these points. Describe the particle’s motion.

Solution:

The path of the particle is given by the parametric equations: x(t) = cos^2(t), y(t) = cos(t), where (t) is the time and (t in R+).

To sketch the path, we can eliminate the parameter t by noting that x(t) = y(t)^2. This is the equation of a part of a parabola that lies to the right of Oy axis, with the vertex tangent to it at the origin. Moreover, the parabola is symmetric with respect to the Ox axis.  However, as cos(t) only takes values from [-1,1], we only get a part of the parabola, where y ranges from [-1,1] and x ranges from [0,1].

The velocity of the particle at time t is given by the derivative of the position function: v(t) = r'(t) = (-2cos(t)sin(t), -sin(t)). Setting each component of the velocity equal to zero gives the times when the velocity is zero:

-2cos(t)sin(t) = 0 => t = 0, pi, 2pi, …

-sin(t) = 0 => t = 0, pi, 2pi, … . So, the velocity is zero at (t = 0, pi, 2pi, …).

The acceleration of the particle at time (t) is given by the derivative of the velocity function: a(t) = v'(t) = (2sin^2(t) - 2cos^2(t), -cos(t))

Substituting the times when the velocity is zero into the acceleration function gives the acceleration at these points: a(0) = a(pi) = a(2pi) = … = (0, -1). So, the acceleration is 0 at (0, -1), where t = 0, pi, 2pi, … .

The particle’s motion can be described as follows: the particle moves along a parabolic path opening to the right, with the vertex at the origin. The particle’s speed decreases until it stops momentarily at (t = 0, pi, 2pi, …), then it changes direction and increases its speed. The acceleration of the particle is always directed downwards.